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The Inverse Laplace Transform

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Definition

If the Laplace transform of a function $F(t)$ is $f(s)$ i.e $L\{F(t)\} = f(s)$ then $F(t)$ is called an Inverse Laplace transform of $f(s)$

We also write $F(t) = L^{-1}\{f(s)\}$

L^{-1} is called the inverse Laplace transformation operator.

Example $L\{t^n\} = \dfrac{n!}{s^{n+1}}$

$$\therefore t^n = L^{-1}\left\{\frac{n!}{s^{n+1}}\right\}$$

Linear property

Theorem If $f_1(s)$ and $f_2(s)$ are Laplace transform of $F_1(t)$ and $F_2(t)$ respectively, then

$$L^{-1}\{c_1 f_1(s) + c_2 f_2(s)\} = c_1 L^{-1}\{f_1(s)\} + c_2 L^{-1}\{f_2(s)\}$$

where c_1 and c_2 are any constants.

Proof we know that

$$L\left[c_1\{F_1(t)\} + c_2\{F_2(t)\}\right] = c_1 L\{F_1(t)\} + c_2 L\{F_2(t)\}$$

$$= c_1 f_1(s) + c_2 f_2(s)$$

$$\Rightarrow c_1 F_1(t) + c_2 F_2(t) = L^{-1}\{c_1 f_1(s) + c_2 f_2(s)\}$$

or $c_1 L^{-1}\{f_1(s)\} + c_2 L^{-1}\{f_2(s)\} = L^{-1}\{c_1 f_1(s) + c_2 f_2(s)\}$

Proved.

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Inverse Laplace transforms of some standard functions.

(1) $L^{-1}\left(\frac{1}{s}\right) = 1$

(2) $L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{\lfloor n-1}$

(3) $L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$

(4) $L^{-1}\left(\frac{s}{s^2-a^2}\right) = \cosh at$

(5) $L^{-1}\left(\frac{1}{s^2-a^2}\right) = \frac{1}{a} \sinh at$

(6) $L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{1}{a} \sin at$

(7) $L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$

(8) $L^{-1} F(s-a) = e^{at} f(t)$

(9) $L^{-1} \frac{s}{(s^2+a^2)^2} = \frac{1}{2a} t \sin at$

(10) $L^{-1} \frac{1}{(s^2+a^2)^2} = \frac{1}{2a^3} (\sin at - at \cos at)$

(11) $L^{-1} \frac{s^2-a^2}{(s^2+a^2)^2} = t \cos at$

(12) $L^{-1} \frac{s^2}{(s^2+a^2)^2} = \frac{1}{2a} \left[\sin at + at \cos at\right]$

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Some problems on Inverse Laplace Transform

Find inverse Laplace transform of the following

Find (i) $\frac{1}{s-2}$

(ii) $\frac{1}{s^2-9}$

(iii) $\frac{s}{s^2-16}$

(iv) $\frac{1}{s^2+25}$

(v) $\frac{s}{s^2+9}$

(vi) $\frac{1}{(s-2)^2+1}$

(vii) $\frac{s-1}{(s-1)^2+4}$

(viii) $\frac{1}{(s+3)^2-4}$

(ix) $\frac{s+2}{(s+2)^2-25}$

(x) $\frac{1}{2s-7}$

Ans of (i) $L^{-1}\left(\frac{1}{s-2}\right) = e^{2t}$

(ii) $L^{-1}\left(\frac{1}{s^2-9}\right) = \frac{1}{3} \sinh 3t$ Ans

(iii) $L^{-1}\left(\frac{s}{s^2-16}\right) = L^{-1}\left(\frac{s}{s^2-4^2}\right) = \cosh 4t$ Ans

(iv) $L^{-1}\left(\frac{1}{s^2+25}\right) = L^{-1}\left(\frac{1}{s^2+5^2}\right) = L^{-1}\left\{\frac{1}{5}\left(\frac{5}{s^2+5^2}\right)\right\}$

$= \frac{1}{5} \sin 5t$ Ans

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(v) $L^{-1}\left(\frac{s}{s^2+9}\right) = L^{-1}\left(\frac{s}{s^2+3^2}\right) = \cos 3t$

(vi) $L^{-1}\left\{\frac{1}{(s-2)^2+1}\right\} = e^{2t}\sin t$ Ans

(vii) $L^{-1}\left\{\frac{s-1}{(s-1)^2+4}\right\} = e^{t}\cos 2t$ Ans

(viii) $L^{-1}\left\{\frac{1}{(s+3)^2-4}\right\} = L^{-1}\left\{\frac{1}{(s+3)^2-2^2}\right\}$

$= \frac{1}{2} e^{-3t} \sinh 2t$ Ans

(ix) $L^{-1}\left\{\frac{s+2}{(s+2)^2-25}\right\} = L^{-1}\left\{\frac{s+2}{(s+2)^2-5^2}\right\}$

$= e^{-2t}\cosh 5t$ Ans

(x) $L^{-1}\left\{\frac{1}{2s-7}\right\} = L^{-1}\left\{\frac{1}{2\left(s-\frac{7}{2}\right)}\right\}$

$= \frac{1}{2} e^{7/2\, t}$ Ans